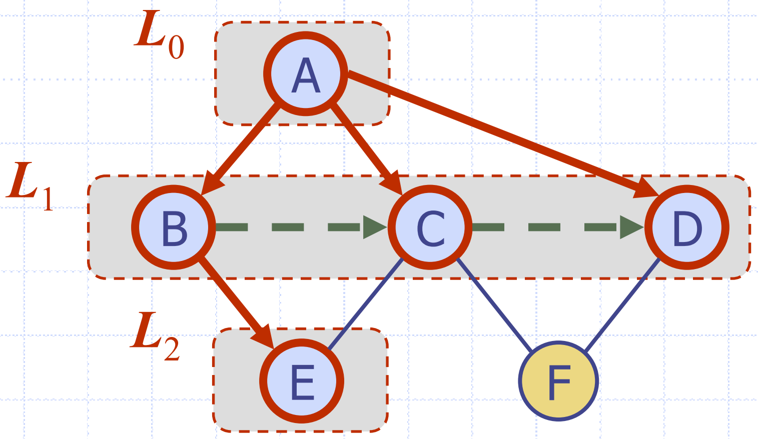
Breadth-First Search



Outline and Reading

- Breadth-first search (§6.3.3)
  - Algorithm
  - Example
  - Properties
  - Analysis
  - Applications
- DFS vs. BFS (§6.3.3)
  - Comparison of applications
  - Comparison of edge labels

Breadth-First Search

- Breadth-first search (BFS) is a general technique for traversing a graph
- A BFS traversal of a graph G
  - Visits all the vertices and edges of G
  - Determines whether G is connected
  - Computes the connected components of G
  - Computes a spanning forest of G
- BFS on a graph with n vertices and m edges takes $O(n + m)$ time
- BFS can be further extended to solve other graph problems
  - Find and report a path with the minimum number of edges between two given vertices
  - Find a simple cycle, if there is one

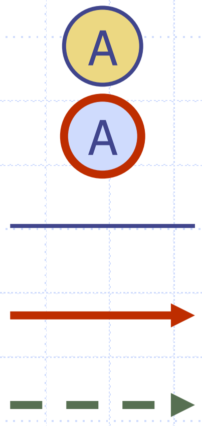
BFS Algorithm

- The algorithm uses a mechanism for setting and getting “labels” of vertices and edges

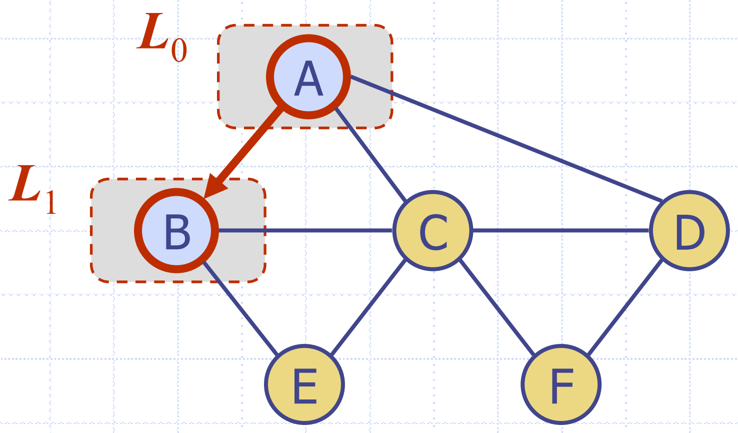
```
Algorithm BFS(G)
    Input graph G
    Output labeling of the edges
        and partition of the
        vertices of G
  for all $u \in G.vertices()$
    $setLabel(u, UNEXPLORED)$
  for all $e \in G.edges()$
    $setLabel(e, UNEXPLORED)$
  for all $v \in G.vertices()$
    if $getLabel(v) = UNEXPLORED$
        $BFS(G, v)$
```

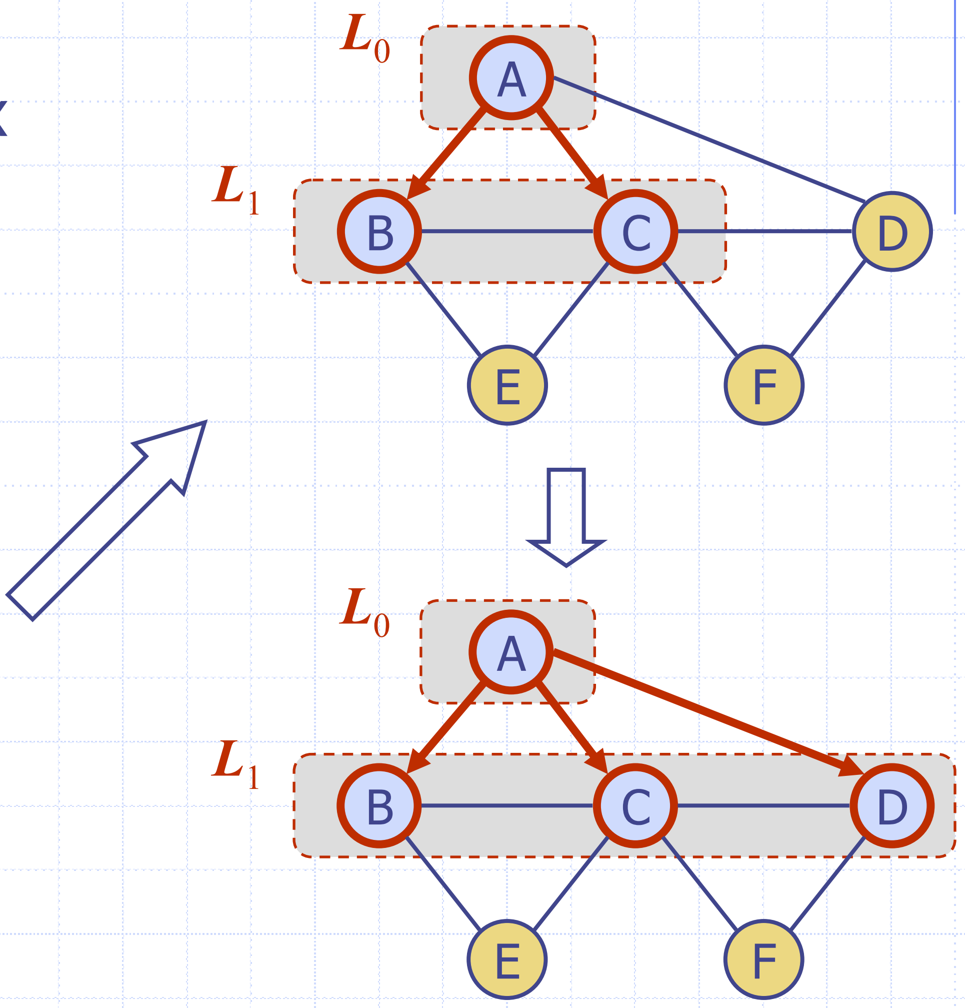
```
Algorithm BFS(G, s)
  $L_0 \leftarrow$ new empty sequence
  $L_0.insertLast(s)$
  $setLabel(s, VISITED)$
  $i \leftarrow 0$
  while $\neg L_i.isEmpty()$
    $L_{i+1} \leftarrow$ new empty sequence
    for all $v \in L_i.elements()$
      for all $e \in G.incidentEdges(v)$
        if $getLabel(e) = UNEXPLORED$
          $w \leftarrow opposite(v,e)$
          if $getLabel(w) = UNEXPLORED$
            $setLabel(e, DISCOVERY)$
            $setLabel(w, VISITED)$
            $L_{i+1}.insertLast(w)$
          else
            $setLabel(e, CROSS)$
    $i \leftarrow i + 1$
```

Example

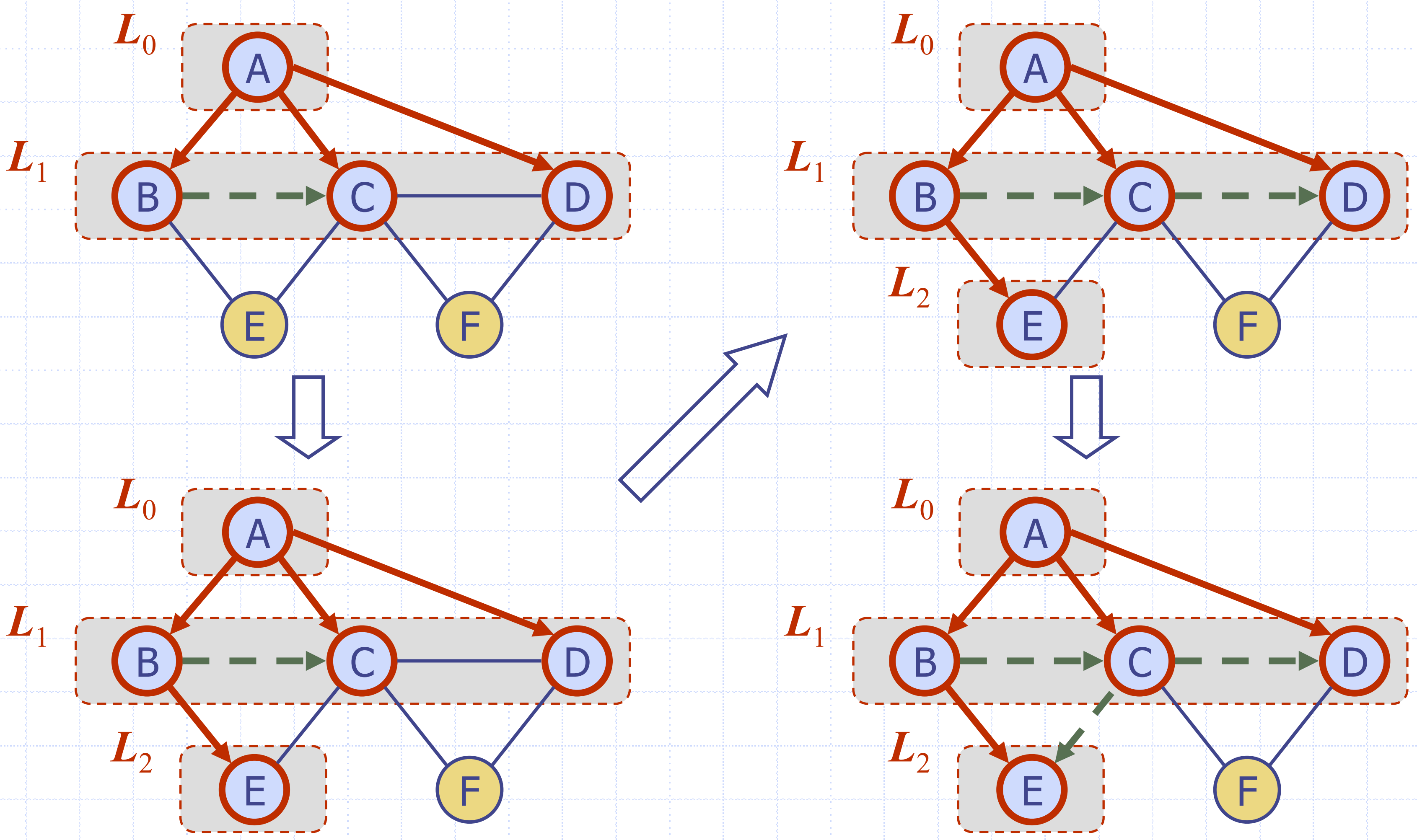


A	unexplored vertex
A	visited vertex
—	unexplored edge
→	discovery edge
- - →	cross edge

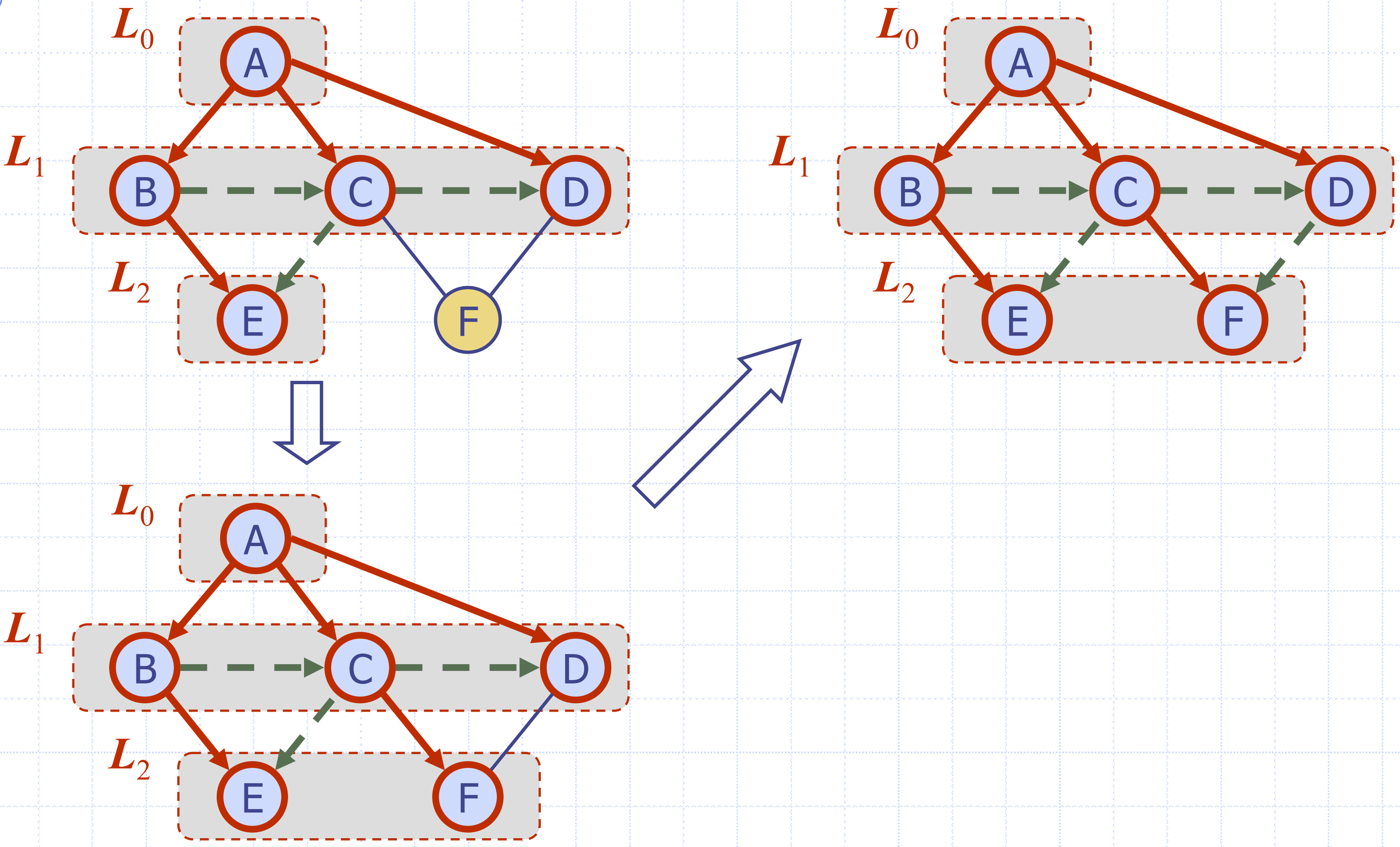




Example (cont.)



Example (cont.)



Properties

Notation

G_s: connected component of s

Property 1

$BFS(G, s)$ visits all the vertices and edges of G_s

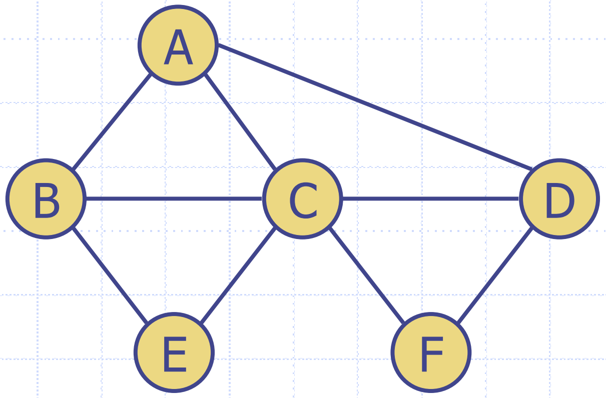
Property 2

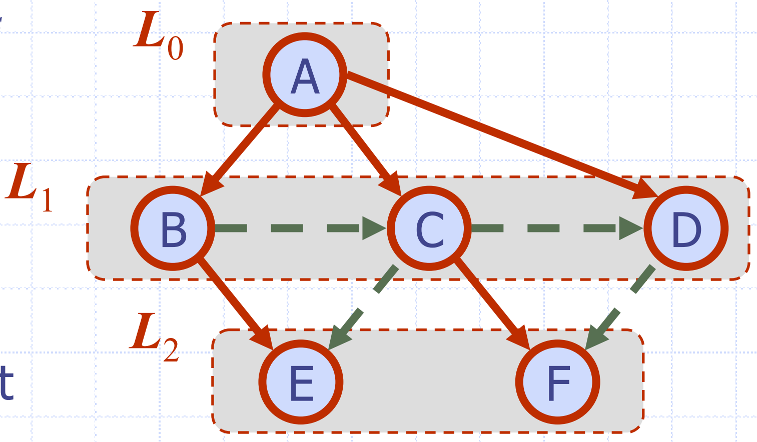
The discovery edges labeled by $BFS(G, s)$ form a spanning tree T_s of G_s

Property 3

For each vertex v in L_i

- The path of T_s from s to v has i edges
- Every path from s to v in G_s has at least i edges





Analysis

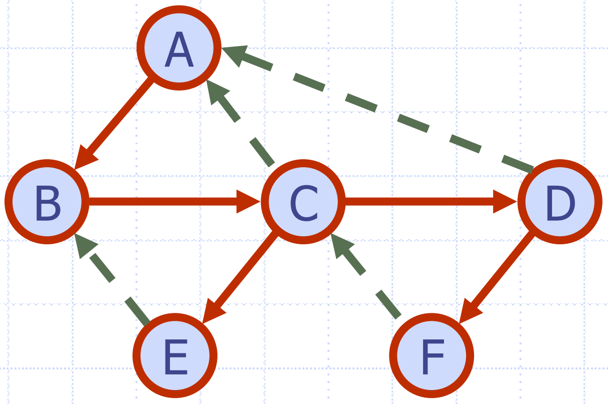
- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- Each vertex is inserted once into a sequence L_i
- Method incidentEdges is called once for each vertex
- BFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$

Applications

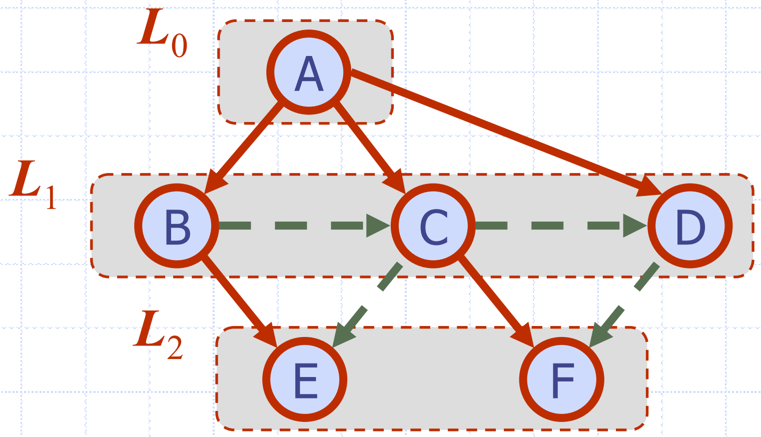
- Using the template method pattern, we can specialize the BFS traversal of a graph G to solve the following problems in $O(n + m)$ time
  - Compute the connected components of G
  - Compute a spanning forest of G
  - Find a simple cycle in G, or report that G is a forest
  - Given two vertices of G, find a path in G between them with the minimum number of edges, or report that no such path exists

DFS vs. BFS

Applications	DFS	BFS
Spanning forest, connected components, paths, cycles	√	√
Shortest paths		√
Biconnected components	√	



DFS



BFS

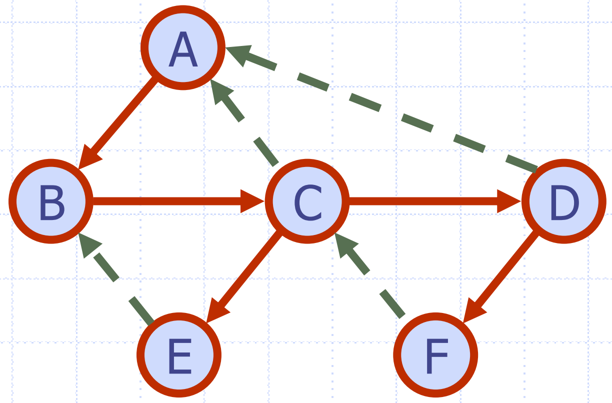
DFS vs. BFS (cont.)

Back edge (v,w)

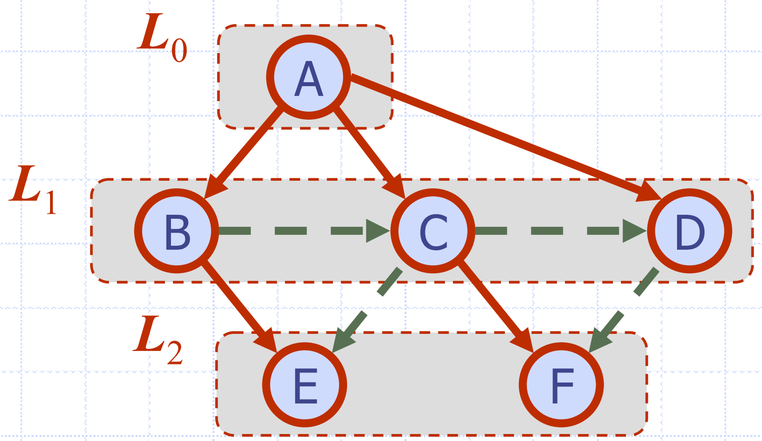
- w is an ancestor of v in the tree of discovery edges

Cross edge (v,w)

- w is in the same level as v or in the next level in the tree of discovery edges



DFS



BFS