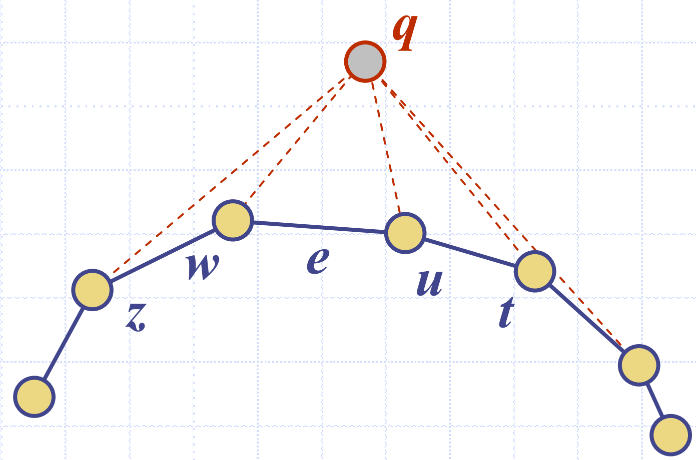
Incremental Convex Hull

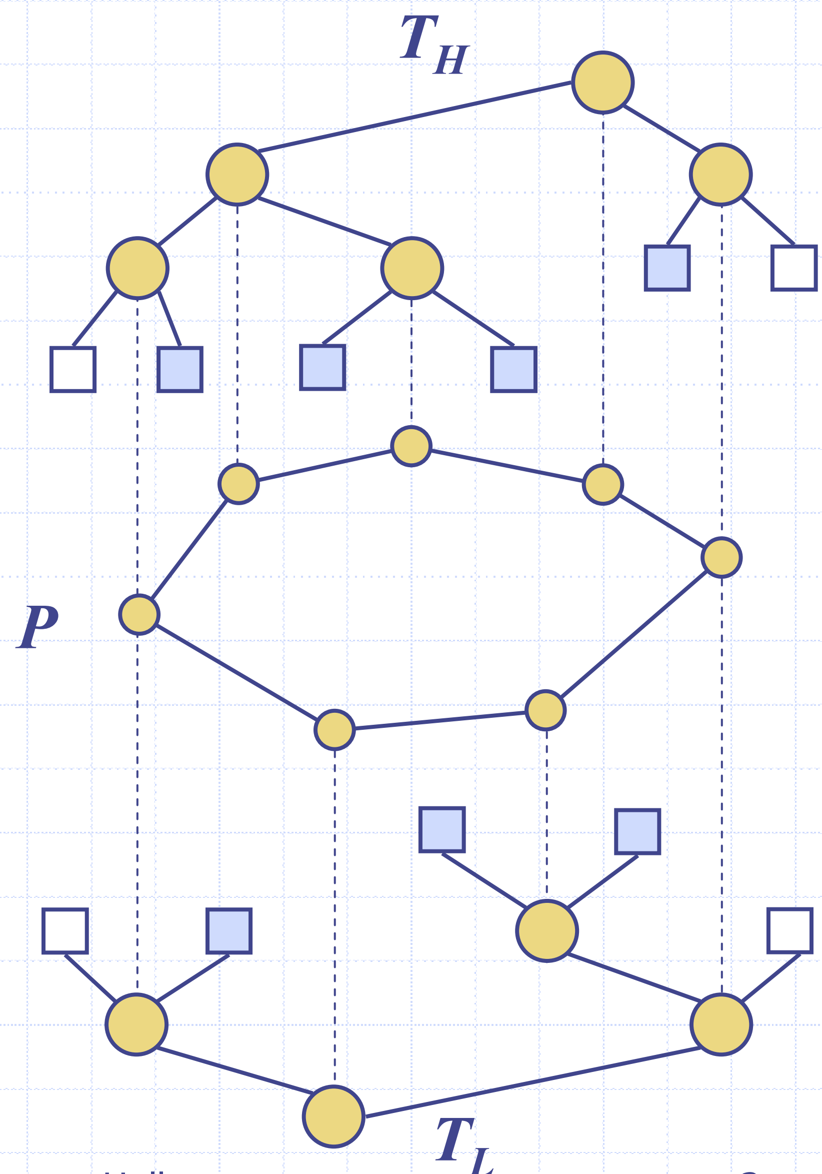


Outline and Reading

- Point location
  - Problem
  - Data structure
- Incremental convex hull
  - Problem
  - Data structure
  - Insertion algorithm
  - Analysis

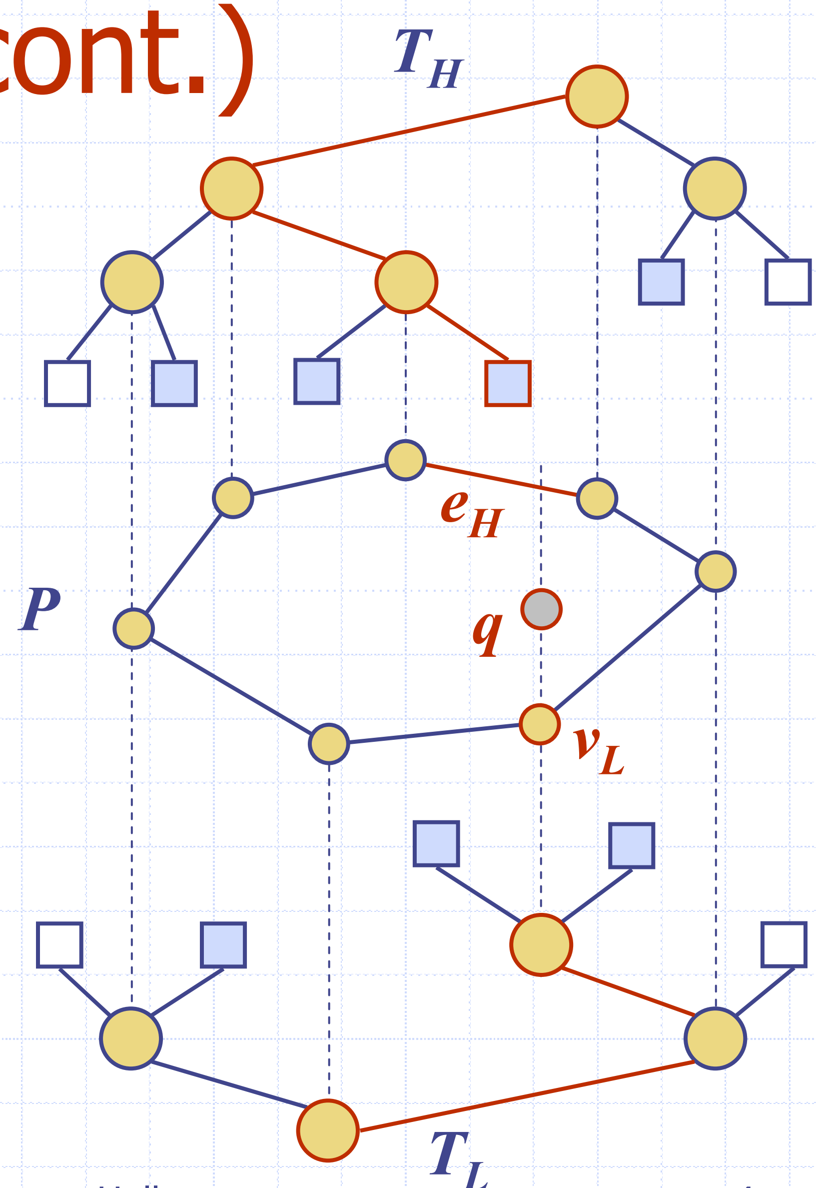
Point Location

- Given a convex polygon P, a point location query *locate*(q) determines whether a query point q is inside (IN), outside (OUT), or on the boundary (ON) of P
- An efficient data structure for point location stores the top and bottom chains of P in two binary search trees, T_L and T_H of logarithmic height
  - An internal node stores a pair $(x(v), v)$ where v is a vertex and $x(v)$ is its x-coordinate
  - An external node represents an edge or an empty half-plane



Point Location (cont.)

- To perform ***locate***(q), we search for $x(q)$ in T_L and T_H to find
  - Edge e_L or vertex v_L on the lower chain of P whose horizontal span includes $x(q)$
  - Edge e_H or vertex v_H on the upper chain of P whose horizontal span includes $x(q)$
- We consider four cases
  - If no such edges/vertices exist, we return OUT
  - Else if q is on e_L (v_L) or on e_H (v_H), we return ON
  - Else if q is above e_L (v_L) and below e_H (v_H), we return IN
  - Else, we return OUT

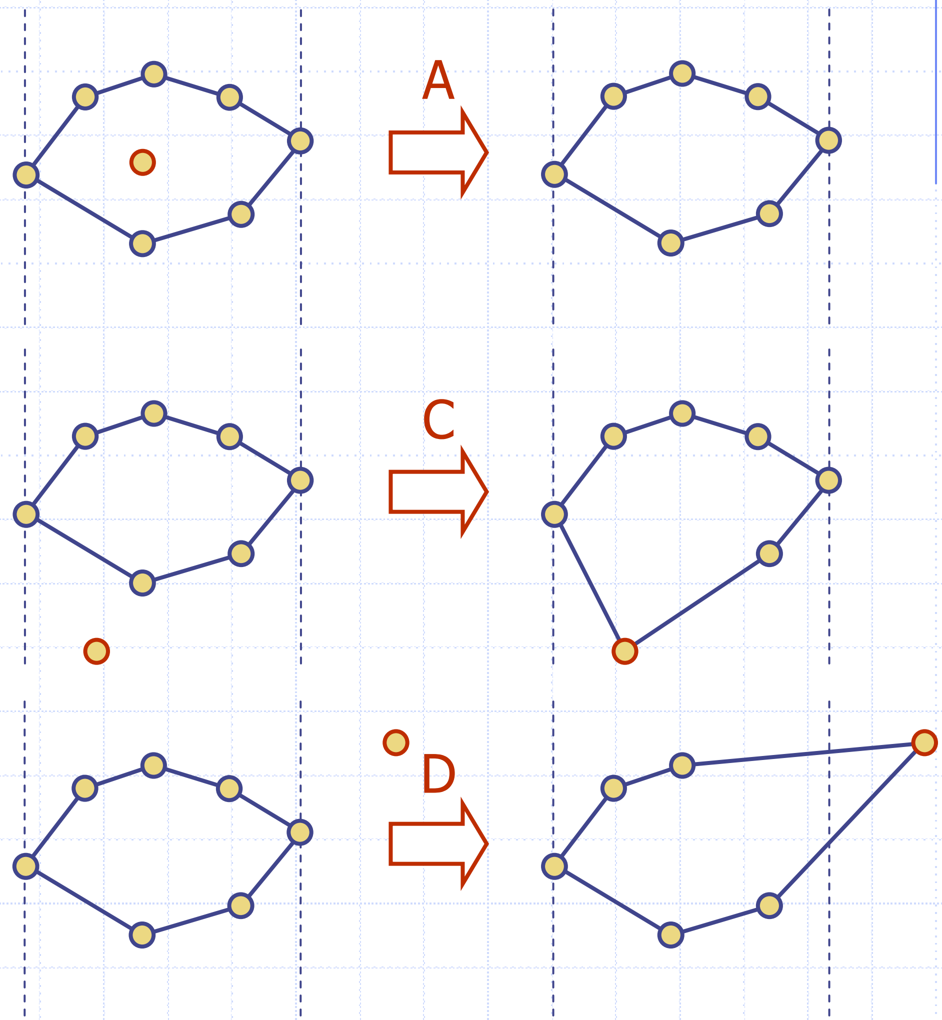


Incremental Convex Hull

- The incremental convex hull problem consists of performing a series of the following operations on a set S of points
  - locate(q): determines if query point q is inside, outside or on the convex hull of S
  - insert(q): inserts a new point q into S
  - hull(): returns the convex hull of S
- Incremental convex hull data structure
  - We store the points of the convex hull and discard the other points
  - We store the hull points in two red-black trees
    - T_L for the lower hull
    - T_H for the upper hull

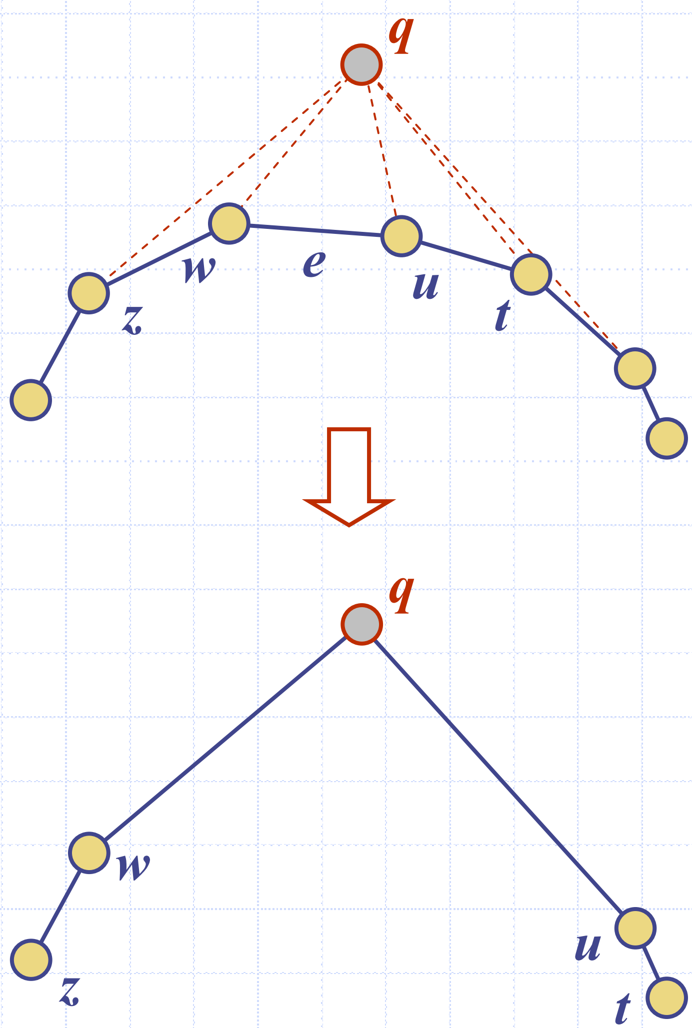
Insertion of a Point

- In operation insert(q), we consider four cases that depend on the location of point q
  - A IN or ON: no change
  - B OUT and above: add q to the upper hull
  - C OUT and below: add q to the lower hull
  - D OUT and left or right: add q to the lower and upper hull



Insertion of a Point (cont.)

- Algorithm to add a vertex q to the upper hull chain in Case B (boundary conditions omitted for simplicity)
  - We find the edge e (vertex v) whose horizontal span includes q
  - $w \leftarrow$ left endpoint (neighbor) of e (v)
  - $z \leftarrow$ left neighbor of w
  - While *orientation*$(q, w, z) =$ CW or COLL
    - We remove vertex w
    - $w \leftarrow z$
    - $z \leftarrow$ left neighbor of w
  - $u \leftarrow$ right endpoint (neighbor) of e (v)
  - $t \leftarrow$ right neighbor of u
  - While *orientation*$(t, u, q) =$ CW or COLL
    - We remove vertex u
    - $u \leftarrow t$
    - $t \leftarrow$ right neighbor of u
  - We add vertex q



Analysis

- Let n be the current size of the convex hull
  - Operation locate takes $O(\log n)$ time
  - Operation insert takes $O((1 + k)\log n)$ time, where k is the number of vertices removed
  - Operation hull takes $O(n)$ time
  - The amortized running time of operation insert is $O(\log n)$