Algorithm Design
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Solution of Exercise C-1.28

Using the two properties of the array, the method is described as follows.

- Starting from element $A[n-1,0]$, we scan A moving only to the right and upwards.
- If the number at $A[i, j]$ is 1, then we add the number of 1s in that column $(i+1)$ to the current total of 1s
- Otherwise we move up one position until we reach another 1.

An example of the traversal of the array is shown in the figure 0.1.

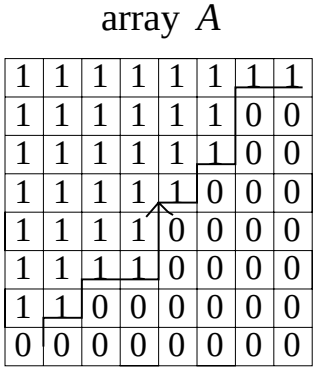

Figure 0.1: The traversal of the array.

The pseudo-code of the algorithm is shown below.

The running time is $O(n)$. In the worst case, you will visit at most $2n-1$ places in the array. In the case that the diagram has all 0s in rows 2 through n and 1s in the first row, then there will be $n-1$ iterations of the **for** loop at constant time (since it will never enter the **while** loop) and 1 iteration of the **while** loop which has n iterations of constant time.

Algorithm NumberOfOnes(A):
Input: An 2D array $n \times n$ A with elements 1s and 0s as described.
Output: The total number of 1s.
$N \leftarrow 0$
$j \leftarrow 0$
for $i \leftarrow n-1$ **to** 0 **do**
  while $j \leq n-1 \wedge A[i,j] = 1$ **do**
    $N \leftarrow N+i+1$
    $j \leftarrow j+1$
return N