

Algorithm Design
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Solution of Exercise C-9.2

The justification is very similar to the argument that the number of iterations in `KMPMatch` is $O(n)$.

Define $k = i - j$ for the sake of analysis. One of the following conditions occurs at each iteration of the loop:

- If $P[i] = P[j]$, then i increases by 1, and k does not change, since j also increases by 1.
- If $P[i] \neq P[j]$ and $j > 0$, then i does not change and k increases by at least 1, since in this case k changes from $i - j$ to $i - f(j - 1)$, which is an addition of $j - f(j - 1)$, which is positive because $f(j - 1) < j$.
- If $P[i] \neq P[j]$ and $j = 0$, then i increases by 1 and k increases by 1, since j does not change.

As a result, the number of iterations is at most $2m$. Therefore, `KMPFailureFunction` runs in $O(m)$ time.