

Algorithm Design
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Solution of Exercise R-8.9

Let $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_n\}$. We can count all possible matchings inductively. There are n ways to match x_1 with vertices in Y . Given this match, there are still $n - 1$ ways to match x_2 with vertices in Y . Given the previous matches, there are still $n - 2$ ways to match x_3 with vertices in Y , and so on. Ultimately, given previous matches, there will be $n - (n - 1) = 1$ way to match x_n . Thus, there are $n!$ possible matches.